**A22** SOLVE LINEAR INEQUALITIES IN ONE **OR TWO** VARIABLE**(S)**, **AND QUADRATIC INEQUALITIES IN ONE VARIABLE;** REPRESENT THE SOLUTION SET ON A NUMBER LINE, **USING SET NOTATION AND ON A GRAPH (higher tier)**

You should be able to solve quadratic equations of the form *ax*2 *+ bx + c* = 0

e.g. *x*2 − 3*x* − 4 = 0 (*x* − 4)(*x* + 1) = 0 *x* = 4 or *x* = −1

e.g. 3*x*2 − 14*x* + 8 = 0 (3*x* − 2)(*x* − 4) = 0 *x* =  or *x* = 4

e.g. *x*2 = 10 − 3*x* *x*2 + 3*x* − 10 = 0 (*x* + 5)(*x* − 2) = 0 *x* = − 5 or *x* = 2

You should also know the shape of a quadratic curve.

If the coefficient of *x*2 is positive, the curve is ‘**smiling**’.

If the coefficient of *x*2 is negative, the curve is ‘**frowning**’.

If f(*x*) > 0 or f(*x*) ≥ 0 we want the values of *x* where f(*x*) is **above** the *x*-axis.

If f(*x*) < 0 or f(*x*) ≤ 0 we want the values of *x* where f(*x*) is **below** the *x*-axis.

**EXAMPLE 1**

Solve *x*2 + 5*x* – 24 ≥ 0

(*x* + 8)(*x* − 3) ≥ 0 First factorise your quadratic expression

Critical values are *x* = −8 and *x* = 3 Solve (*x* + 8)(*x* − 3) = 0

−8 3 *x* Always draw a sketch of your curve

Show where the curve cuts the *x*-axis

by solving (*x* + 8)(*x* − 3) = 0

*x* ≤ −8 and *x* ≥ 3 We want the area where *y* ≥ 0

If you are asked to write the **solution set** of the inequality *x*2 + 5*x* − 24 ≥ 0 then the answer would be: {*x* : *x* ≤ −8, *x* ≥ 3}

**NOTE:** There are TWO regions so we write the answer as TWO inequalities.

**EXAMPLE 2**

Find the solution set of the inequality 6(*x*2 + 2) < 17*x*

6*x*2 + 12 < 17*x* First expand the bracket

6*x*2 − 17*x* + 12 < 0 Rearrange to the form *ax*2 + *bx + c* < 0

(3*x* − 4)(2*x* − 3) < 0 Factorise in order to find where it cuts the *x*-axis

Critical values are *x* = 4/3 and 3/2 Solve (3*x* −4)(2*x* − 3) = 0

  *x* Sketch the curve and shade below the axis

 < *x* <  We want the region where f(*x*) is **below** the *x*-axis

Solution set = { *x* :  < *x* <  } Make sure your answer is given in the correct form

**NOTE:** There is ONE region so we write the answer as ONE inequality.

**EXAMPLE 3**

Solve *x*(*x* + 9) ≤ 0

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*x*(*x* + 9) ≤ 0This is already factorised with 0 on one side so there is no need to expand the brackets

Critical values are *x* = 0 and *x* = −9

−9 0 *x* Sketch the curve and shade below the axis

−9 ≤ *x* ≤ 0 We want the region where f(*x*) is **below** the *x*-axis

There is only one region so write as one inequality

**EXAMPLE 3**

**EXAMPLE 4**

Solve the inequality 14 + 5*x* < *x*2

14 + 5*x* − *x*2 < 0 Rearrange to the form *ax*2 + *bx + c* < 0

(2 + *x*)(7 − *x*) < 0 Factorise in order to find where it cuts the *x*-axis

−2 7 *x* The curve is **'frowning'** as we have −*x*2

*x < −*2 and *x* > 7 We want the region where f(*x*) is **below** the *x*-axis

**OR**

14 + 5*x* − *x*2 < 0 Rearrange to the form *ax*2 + *bx + c* < 0

*x*2 − 5*x* − 14 > 0 Multiply each term by −1 which changes < to >

(*x* + 2)(*x* − 7) > 0 Factorise in order to find where it cuts the *x*-axis

−2 7 *x* The curve is **'smiling'** as we have +*x*2

*x < −*2 and *x* > 7 This method gives the same answer as the 1st method

**EXERCISE:**

1. Solve each of these inequalities.

(a) *x*2 + 9*x* + 18 ≤ 0 (b) *x*2 − *x* − 20 < 0

(c) (*x* − 2)(*x* + 7) > 0 (d) *x*2 − 5*x* ≥ 0

(e) 2*x*2 − 11*x* + 12 < 0 (f) (5 *+ x*)( 1 *−* 2*x*) ≥ 0

(g) 15 + 2*x* – *x*2 ≤ 0 (h) 21 − *x* – 2*x*2 > 0

(i) *x*(5*x* − 2) > 0 (j) *x*2 − 2*x* > 35

2. Find the solution set for each of these inequalities.

(a) *x*2 − 4*x* + 3 ≤ 0 (b) *x*2 + *x* − 42 < 0

(c) *x*(*x* + 2) > 48 (d) 3*x*2 + 14*x* − 5 ≥ 0

(e) 2*x*2 > 11*x* − 12 (f) 16 – *x*2 ≤ 6*x*

(g) 7 + 2(4*x*2 – 15*x*) ≤ 0 (h) *x*2 – 4(*x* + 6) > 8

(i) 3*x*(5 − *x*) > 0 (j) (*x* + 5)2 ≥ 1

3. Solve  ≥ 4*x*

4. Find the solution set for which 15 + 2*x* ≤ *x*2

5. Find the set of values for which 6 + *x* ≥ *x*2 and *x* + 2 < *x*2

6. Find the solution set for (*x* − 3)(2*x* + 3) < 2*x*(1 − 2*x*) − 5

**ANSWERS:**

1. (a) −6 ≤ *x* ≤ −3 (b) −4 < *x* < 5

(c) *x* < −7 or  *x* >2 (d) *x* ≤ 0 or  *x* ≥ 5

(e) < *x* < 4 (f) −5 ≤ *x* ≤ 

(g) *x* ≤ −3 or  *x* ≥ 5 (h) −< *x* < 3

(i) *x* < 0 or  *x* >  (j) *x* < −5 or  *x* > 7

2. (a) { *x* : 1 ≤ *x* ≤ 3 } (b) { *x* : −7 < *x* < 6 }

(c) { *x* : *x* < −8, *x* > 6 } (d) { *x* : *x* ≤ −5, *x* ≥ }

(e) { *x* : *x* < , *x* > 4 } (f) { *x* : *x* ≤ −8, *x* ≥ 2 }

(g) { *x* :  ≤ *x* ≤ } (h) { *x* : *x* < −4, *x* > 8 }

(i) { *x* : 0 < *x* < 5 } (j) { *x* : *x* ≤ −6, *x* ≥ −4 }

3. *x* ≤ 2 or *x* ≥ 6

4. { *x* : *x* ≤ − 3, *x* ≥ 5}

5. −2 ≤ *x* < −1 and 2 < *x* ≤ 3

6. { *x* : −  < *x* < }